Using Fermi Problems to Motivate 4th Grade Primary School Students in Math Lessons

Sinem ABAY¹, Sevil BÜYÜKALAN FİLİZ²

Abstract

The concept of motivation is described as a broad structure that includes all the internal and external conditions that affect how behavior is prompted, maintained and controlled. Math motivation is described as love of mathematics, an interest in it, and a high level of motivation with respect to this lesson, a wish to take part in math lessons and an interest in math both in and out of school. Mathematical problems can be varied as problems with different ways of solving them to routine problems with only one correct answer and open-ended, non-routine problems that every individual can interpret differently and whose outcomes vary from person to person. There is no single correct answer to open-ended problems, and these kinds of problems, which are also called non-routine problems, are important parts of the mathematics teaching program. Furthermore, non-routine problems are forms of problem that require students to use their imagination and that encourage them to think creatively and critically. For this reason, it can be said that these types of problems are educational in that they teach the students how to overcome the problems that they will encounter throughout their lives. Fermi problems can be regarded as examples of open-ended problems. Fermi problems are open-ended, non-routine problems that require students to make systematic guesses by making assumptions before starting on a solution using simple calculations. They are the kinds of problems that can be solved in different ways, that have no single correct answer and that students can only solve by making assumptions. The goal of this study is to influence 4th-grade primary school students' math motivation by using Fermi problems. To this end, a total of 40 4th grade students in a state-run school in the central district of Tokat Province were divided into one test and one control group, and their math motivation levels were measured. Afterwards, a four-week implementation was carried out using the test group during which the students were made to solve Fermi problems. The math motivation levels for both groups were tested again after this implementation was over. The first and last tests were compared in the light of the data obtained. According to the research results, while no change was observed in the math motivation levels of the control group, which was taught mathematics using traditional methods, a development was seen in the math motivation levels of the test group. Therefore, it is suggested that Fermi problems may be used to increase interest and motivation in math classes.

Keywords

Fermi problems
Mathematical motivation
Problem solving

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Introduction

It is seen that in curriculum studies aimed at overcoming the perception of mathematics lesson being a boring and toilsome lesson, skills such as problem solving, critical thinking and creativity are prioritized in line with the constructivist approach. Yücel, Karadağ, and Turan (2013) have stated that although there have been changes and developments in all these elements compared to the past, according to the current questions of PISA, which measures the level of mathematics literacy, and TIMSS, where mathematics success averages are determined, there is still no significant success in this field. With the renewed mathematics curriculum (2005), action was taken in line with the understanding that "every child can learn mathematics". Being active participants of the students in the mathematical process is among the main goals of the new mathematics program. According to Ersoy (2006, p.33), classrooms should be equipped with mathematical materials in order for students to form abstract mathematical thoughts; necessary tools and equipment should be used in activities. Thus, students notice the necessary information by using the model and have the opportunity to examine and solve problems. In the updated mathematics curricula, it was stated that verbal expression of students' thoughts in the mathematics learning-teaching process has an important place in the internalization, understanding and structuring of mathematical concepts, and that students should be encouraged to establish individual and interpersonal communication during the teaching process (MEB, 2018).

The difficulties of primary school students' experiences when dealing with real-world word problems in mathematics are widely known. Many people believe that there is the only one correct way to solve a mathematics problem (NCTM, 2000; cited from Gökbulut, 2006, p. 275). However, for many problems, people can find appropriate reasoning forms and solutions for their own characteristics (Gökbulut, 2006, p. 275). Problem-solving ability is one of the most basic skills required for human to continue their existence. Due to its role in coping with difficulties in every field, one of the main goals of school mathematics programs is related to the development of this ability (Altun, Dönmez, İnan, Taner, and Özdílek, 2001, p. 212). The abstract nature of the problems can be shown among the reasons for students' mathematical failure. Preventing this failure is possible by concretization of abstract problems. For this purpose, problems should be able to be associated with real life and concretized with the help of materials.

The absence of problems which are open-ended, that do not direct the student, that are not routine and that enable students to study by thinking about real life situations shows that the mathematics curriculum has a significant deficiency (Taşova and Delice, 2012, p. 75) The role of problem solving, which is the basic building block of mathematics education, is undeniable at all ages and levels of education. Problem solving, which leaves its mark on the age we are in, is among the aims of all lessons. It should be known that the contribution of problem solving to 21st century teaching situations is great. For this reason, the problem, the structure of the problem solving and increasing the success in problem solving are the topics studied by many educators and psychologists (Kılıç and Samancı, 2005, p.100). This method, which is frequently preferred by mathematics educators in terms of the student's mental development, thinking and questioning, and producing various solutions, is also a way that is referred for providing many gains to the student.

Stating that the problem should not be perceived as a research or question whose solution is previously known, Pesen (2008, p.65) has stated that in order for a mathematical situation to be a problem, the way to reach a solution should not be clear and the student should use his existing knowledge and reasoning skills. Mathematical problem solving; argues that problems should include situations ranging from routine mathematical problems to complex problems whose answers are not immediately apparent, and open-ended research using related mathematical thinking processes (Akay, Soybaş, and Argün, 2006, p.129). Based on these views, having no single solution, being cannot be predicted of its answer immediately, and encouraging the student to question and think creatively can be considered as the basic characteristics that should be found in a mathematical problem.

Mathematical problems can be diversified as routine problems with different solutions and a single correct answer, and open-ended non-routine problems, where each individual can make a different interpretation and its result may vary from person to person. Open-ended problems do not have a single correct answer and such questions, which are also called non-routine problems, are the
most important parts of the mathematics curriculum (Keskin, 2008, p. II). By pointing out that not making students solve problems other than routine problems is an “unforgivable mistake” and doing so deprives students of “imagination and judgment”, Polya shows the importance he attaches to non-routine problems (Yazgan and Bintaş, 2005, p. 211). Routine problems, also called closed problems, are those that are clearly formulated and well-structured in terms of tasks, in which the correct answer can be determined in some simple ways and the necessary information is given in the problem statement (Akay, Soybaş, Argün, 2006, p. 132).

While students are trying to solve non-routine problems, they do not use the operations rotely, they learn to use them because the operations are required by problems (Olkun, Şahin, Akkurt, Dikkartin, Gülbağcı, 2009, p.67). Based on this statement, it can be concluded that non-routine problems are types of problem that requires the student to use their imagination and leads them to creative and critical thinking. For this reason, it can be said that such problems are educational tools for students to overcome the problems they will face throughout their lives. Non-routine problems are also called open-ended problems. In open-ended problems, since there is no formulation that guarantees a correct and complete solution, such problems are also referred to as poorly structured problems (Akay, et al., 2006, p.133).

The Fermi Problems, put forward by the famous physicist Enrico Fermi (1938), are examples of non-routine problems. Fermi problems involve making assumptions, making estimations about quantities that are unlikely to be calculated with a systematic way of thinking and limited information (Arleback, 2009, p.332). Taplin (2007) has explained Fermi problems as follows: “It is the type of problem that encourages students to be more creative and that enough information is not given. When people see Fermi Problems, they first think they need more information to solve them. In fact, although common ideas and experiences make acceptable solutions possible, the solution of these problems is based on the sum of the knowledge and experiences of the students. These problems are not deterrent and can be solved in a collaborative environment” (Hıdıroğlu, 2012, p.44).

According to Arleback (2009), Fermi problems are open-ended, non-routine problems that require students to make systematic predictions and making assumptions before starting solutions with simple calculations. Below are a few questions that may serve as examples of Fermi problems:

- How much money in total is spent in your school canteen in one day?
- How many liters of water on average is used in your home in a week?
- What is the total distance you have walked in a year?
- How many kilos of garbage does a family throw out on average in a year?

As it can be understood from the examples, Fermi problems are the types of problems that are solved based on assumptions and do not have a single correct result, which lead students to question, think and creativity. The purpose of solving these problems is to enable the student to think multifaceted and to obtain as many assumptions as possible. In this context, it would be appropriate to use this type of problem to increase the interest and motivation for mathematics lesson. It can be said that students may be able to see the fun side of mathematics and realize that this lesson is not just about reaching correct results.

Loving, understanding and learning mathematics starts with getting to know it correctly, above all. If mathematics is a support that facilitates our life, paves us the way for logical and rational thinking to deal with the problems that we encounter at every moment in our daily life, enables us to evaluate events more consistently and objectively, and makes our lives colorful and fun, trying to understand it becomes a responsibility rather than a choice (Yenilmez and Can, 2006, p. 48). Motivation, which determines the amount of energy that an individual can directly spend to achieve their goal (Ginsberg and Wlodkowski, 2009), is one of the important components affecting learning (Cahi, 2009; cited from Balantekin, Oksal, 2014, p. 103). Martin and Briggs (1986; cited from Yaman, Dede, 2007, p.616) define motivation as a broad structure that includes all of the internal and external conditions that affect the arousal, maintenance and control of behavior. The concept of motivation has been included in the literature as internal and external. External motivation arises as a result of the effect of external reinforcers. Studying their lesson or fulfilling a given task of a student in order to
avoid the reaction of the teacher or to gain his/her praise and appreciation can be an example of this. Internal motivation, on the other hand, is the individual’s reactions to internal needs. The need to be competent, to know, to understand can be given as an example of this (Akbaba, 2006, p. 345). The concept of mathematical motivation, which corresponds to the situations of liking and being interested in mathematics, having a high level of motivation towards this lesson, the desire to participate in the mathematics lesson, and being related to mathematics at school and outside of school, has been examined and researched by many researchers in the literature. Some of the studies on measuring motivation in the relevant literature are summarized below:

In the study conducted by Yaman and Dede (2007) the difference in the motivation of secondary school students towards mathematics and science and technology lesson according to gender, grade level and favorite course variables has been examined. For this A Questionnaire for Motivation toward Science Learning developed by Dede and Yaman has been used. The questionnaire has been applied to 740 students who were selected on a voluntary basis and were studying at 6th, 7th and 8th grades in secondary school in Sivas city center in the 2005-2006 academic year. As a result of the analysis of the data, it has been determined that the motivation levels of secondary school students differ significantly according to gender, grade level and favorite lessons. In addition, when the analysis results have been examined, it has been seen that the motivation groups were correctly separated at medium level in terms of predicted variables.

In the study conducted by Balantekin and Oksal in 2014, it has been aimed to develop a scale to determine the motivation levels of primary school 3rd and 4th grade students in mathematics lesson. The draft scale, prepared in a five-point Likert style and consisting of 41 items, has been applied to 308 students. As a result of this study conducted to determine the validity and reliability of the scale, it has been seen that the scale was at an acceptable level in terms of validity and reliability. In the study of Tahirıoğlu and Çakır in 2014, it has been aimed to develop a valid and reliable measurement tool that measures the motivation of primary school 4th grade students towards learning mathematics. During the scale development process, a draft scale has been prepared according to the 5-point Likert-type agreement degree. After the changes made in line with the opinions of the experts, the scale has been applied to 67 primary school 4th grade students. Analysis results have shown that the scale was sufficiently valid and reliable.

In this study, it was aimed to affect the motivation of primary school 4th grade students towards mathematics lesson by using Fermi problems. For this purpose, solutions were also sought for the following sub-problems;

1. What is the motivation level of the experimental group towards the mathematics lesson before the application?
2. What is the motivation level of the experimental group towards the mathematics lesson as a result of the application?
3. Is there a significant difference between the pre-test results before the application and the post-test results after the application of the control group?
4. Is there a significant difference between the pre-test results before the application and the post-test results after the application of the experimental group?

Mathematics education provides individuals with a wide range of knowledge and skills to help them understand the physical world and social interactions. It also facilitates creative thinking and provides aesthetic development. In addition, it accelerates the development of individuals' reasoning skills by creating environments where various mathematical situations are examined (Gökbulut, Yangın and Sidekli, 2008; p. 214). The acquisition of these skills is possible by using the variety of methods used in mathematics teaching, associating them with daily life and using real life problems. Fermi problems include these gains (Yanbıyık, 2016). Non-routine problems that have been tried to be applied in primary school in recent years encourage the student to be creative and think critically. Based on the assumption that these problems are applied effectively, it is thought that the acquisition of targeted skills will be facilitated by popularizing the use of Fermi problems. At the same time, an increase in students’ interest and motivation in mathematics can be ensured. Based on this situation, it
reveals the importance of this study, which is aimed to examine whether Fermi problems affect the motivation towards mathematics lesson in primary school 4th grade.

Method

In this study, which aims to examine the effect of Fermi problems on motivation towards mathematics lesson in primary school 4th grade students, an experimental design with pretest posttest experimental-control groups was used. The experimental design is defined as research designs that are used to explore cause-and-effect relationships between variables.

Study Group

The study group of the study consists of a total of 40 students from the 4th grade students of a state school from the central primary schools of Tokat, from two classes with similar academic achievements. In the experimental group, there were a total of 18 students, including 10 girls and 8 boys, and in the control group, there were a total of 22 students, including 14 girls and 8 boys. Homogeneous sampling method, one of the purposeful sampling methods, was used in the selection of the study group. The purposeful sampling method is to select information-rich situations in the context of the purpose of the study in order to conduct in-depth research. Homogeneous sampling, on the other hand, is the creation of the sample from a similar subgroup or situation in the universe related to the problem of the research (Büyüköztürk, Çakmak, Akgün, Karadeniz, and Demirel, 2014, p. 91).

Data Collection Tool

In the data collection process of the research, a Likert type scale developed by Dede and Yaman (2008) was used to measure the motivation of primary school 4th grade students towards mathematics lesson. During the data collection process, after the application of the scale to the experimental and control groups, Fermi problems were solved with the experimental group in one lesson each week, and the control group continued their mathematics lessons with traditional methods. At the end of a total of 4-week application period, the motivation scale was applied to the students again.

Before the data collection process, a pilot study was conducted with a different group to solve the Fermi problem. After the pilot application, the experimental group was divided into small groups, the week's problem was written on the board, and then the groups were allowed to solve the problem for one lesson hour. The researcher served as a guide in the solution of the questions and did not interfere with the solution of the questions. The Fermi problems addressed to students in the lessons in which the applications were made are as follows:

- How many vehicles are there on a 500 m road?
- What is the total weight of all students in your school in kg?
- How many liters of water is spent in your home a month?
- How much money in total is spent in your school canteen in one day?

Each of the questions was solved by the groups under the guidance of the researcher in a total of 160 minutes, covering a mathematics lesson on a day of each week. After 4-week Fermi problem solutions, the motivation scale was applied to the students in the experimental and control groups. The effect of Fermi problems on motivation towards mathematics lesson was examined by comparing the pre-test and post-test results obtained. In the process, what expected from the student is to act with assumptions for the solution of the Fermi problem, to think about as many possibilities as possible, to make calculations with logical inferences and to reach the result of the problem. In addition, since the aim of the research was to affect motivation, whether the students found the answer was not taken into account. Below are a few examples of the students' solution to Fermi problems in order to create a mental scheme for the application process:
In Figure 1, in line with the answer given to the question "How much money in total is spent in your school canteen in one day?", it was seen that students calculated the possible prices of all food and beverages sold in the school canteen and also calculated how many of these products were sold and act in accordance with the solution of Fermi problems.

In Figure 2, in line with the answers given to the question "How many vehicles are there on a 500 m road?", all possible vehicles that can be found on a highway were written and their lengths were estimated. Afterwards, drawings were made by modeling and necessary calculations were made.

In Figure 3, in line with the answers given to the question "What is the total weight of all students in your school in kg?", the number of students and teachers in the school was calculated,
other employees were not included. In this case, only the possible weights of teachers and students were calculated and the result was reached.

In Figure 4, in line with the answers given to the question "What is the total weight of all students in your school in kg?", the students estimated their possible weight by including all the staff and students in the school building. As a result of the obtained predictions, they reached the solution of the Fermi problem.

**Data Analysis**

The pre-test and post-test frequency distributions in the collected data were determined using the SPSS 18 package data program and the motivations of the experimental and control groups determined from primary school 4th grade students towards mathematics lesson were examined. After the application to affect the motivation of the experimental group towards the mathematics lesson, the data obtained from the measurement tool applied to both groups were compared with the t test application. The results obtained were interpreted in terms of the effect of Fermi problems on motivation towards mathematics lesson.

**Findings**

The findings obtained from the data collection process of the study were interpreted in line with the sub-problems.

1. **Findings Regarding the Experimental Group’s Mathematical Motivation Level Before the Application**

According to the findings obtained before the application, information about the average, maximum and minimum values of the scores obtained by the experimental group from the scale consisting of 23 items were given in Table 1:

<table>
<thead>
<tr>
<th>Success Score</th>
<th>N</th>
<th>Minimum Score</th>
<th>Maximum Score</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td></td>
<td>1.39</td>
<td>4.09</td>
<td>3.4239</td>
</tr>
</tbody>
</table>

When Table 1 is examined, it is seen that the experimental group obtained minimum 1.39 and maximum 4.09 points from the scale applied. In addition, the average score obtained by the students in the experimental group from this test was measured as 3.4239.

2. **Findings Regarding the Experimental Group’s Mathematical Motivation Level After the Application**

According to the findings obtained after the application, information about the average, maximum and minimum values of the scores obtained by the experimental group from the scale consisting of 23 items were given in Table 2:
Table 2: Average Scores of the Experimental Group after the Application

<table>
<thead>
<tr>
<th>Success Score</th>
<th>Minimum Score</th>
<th>Maximum Score</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>2.96</td>
<td>5.00</td>
<td>4.3652</td>
</tr>
</tbody>
</table>

When Table 2 is examined, it is seen that the experimental group obtained minimum 2.96 and maximum 5.00 points from the measurement tool applied. In addition, the average score obtained by the students in the experimental group from this test was measured as 4.3652.

3. Findings Regarding Pre Test-Post Test Results of Control Group

The information on whether there is a significant difference between the scores of the control group obtained from the scale before and after the traditional mathematics lessons was given in Table 3:

Table 3: Pre Test-Post Test Results Applied to the Control Group

<table>
<thead>
<tr>
<th>N: 20</th>
<th>N</th>
<th>Pre-test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-test</td>
<td>Post-test</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematical Motivation</td>
<td>22</td>
<td>3.59</td>
<td>0.80</td>
</tr>
</tbody>
</table>

* p>0.05

When Table 3 is examined, no significant difference is seen between the pre-test and post-test scores of motivation of students in the control group towards mathematics lesson (t(19)= - 0.433, p>0.05). When the average scores of the students according to the application performed using traditional methods were examined, it was seen that the pre-test scores were $\bar{x}$ = 3.59, and the post-test scores were $\bar{x}$ = 3.64. This result can be interpreted as that the traditional methods applied on the control group did not have any effect on increasing students' motivation level towards mathematics lesson.

4. Findings Regarding Pre-Test-Post-Test Results of the Experimental Group

Information on whether there is a significant difference between the scores of the experimental group obtained from the scale before and after the application was given in Table 4:

Table 4: Pre Test-Post Test Results Applied to Experimental Group

<table>
<thead>
<tr>
<th>N: 20</th>
<th>N</th>
<th>Pre-test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-test</td>
<td>Post-test</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematical Motivation</td>
<td>18</td>
<td>3.42</td>
<td>0.69</td>
</tr>
</tbody>
</table>

* p<0.05

When Table 4 is examined, no significant difference is seen between the pre-test and post-test scores of motivation of students in the control group towards mathematics lesson (t(19)= - 5.701, p<0.05). When the average scores of the students according to the application performed using Fermi problems were examined, it was seen that the pre-test scores were $\bar{x}$ = 3.42, and the post-test scores were $\bar{x}$ = 4.36. The increase in the average motivation towards mathematics lesson reveals that Fermi problems are effective in increasing the mathematical motivation level of students.

Discussion, Conclusion, and Recommendations

Fermi problems appear as a type of problem that is studied but has a low application rate in teaching environments. It can be said that supporting non-routine problems and using them in learning environments has become a matter in our country and in the world, however, Fermi problems are not a type of problem that is frequently preferred in classroom environments, educational environments, or scientific studies. When the literature was reviewed, it was seen that the studies on Fermi problems were associated with mathematical modeling skills, and not aimed at interest and motivation towards mathematics (Peter-Koop (2005), Ärlebäck (2009), Abay and Gökbulut (2017)). In this study, applications were made using Fermi problems after the motivation scale applied to two different groups, and at the end of the process, it was seen that mathematics lessons with Fermi problems increased students' motivation towards mathematics.
In Peter-Koop's (2005) study titled "Fermi Problems in Primary School Mathematics Classrooms: Interactive Modeling Processes of Students", Fermi Problems have been directed to 3rd and 4th grade students in order to improve their modeling processes and solution analyses has been made. As a result of the study, it has been observed that many students could not develop a solution plan and a positive development was observed in modeling processes. Considering the use of Fermi Problems in the development of modeling processes, and the state of increasing the motivation level towards the lesson in this study, it can be said that the study findings are parallel.

In his study titled "Using Fermi Problems in the Application of Mathematical Modeling", Ärlebäck (2009) has used Schoenfeld's Mathematical Modeling Diagram (MAD) to apply mathematical modeling to students above secondary education level. Ärlebäck has directed Fermi Problems to three groups of students and made their solution analysis using the Mathematical Modeling Diagram. Mathematical models are frequently encountered in students' solutions. The use of Fermi Problems in the application process of this study is similar to the application process of this study, and there is no similarity in terms of findings.

In their study conducted in 2017, Abay and Gökbulut aimed to describe the mathematical modeling situations of classroom teacher candidates in Fermi problems, and for this purpose, classroom teacher candidates were solved Fermi problems. Considering the obtained results, it was seen that the mathematical modeling skills of the classroom teacher candidates in Fermi problems were not at a sufficient level. The use of Fermi Problems in the process of determining the mathematical modeling skill level is similar to the application process of this study.

When the literature is reviewed, there is no scientific study encountered in which Fermi problems are used to provide or increase motivation. In this context, it can be suggested that researchers should conduct research using Fermi problems in order to increase interest in mathematics or to improve mathematical attitude. According to the results of the study, the following suggestions were made to the curriculum experts, teachers and researchers:

- It can be suggested that while preparing education and training programs, the inclusion of Fermi problems in the mathematics curriculum of primary school 3rd and 4th grades in terms of gaining creativity, critical thinking and problem solving skills and associating mathematics with daily life,
- Adding Fermi problem solving practices to the applications of teachers in order to increase interest in mathematics lesson,
- Researchers should work on the effects of Fermi problems on students' academic achievement, mathematical attitude, and the association of mathematics with daily life.

References


